Technical Summary, PhD Thesis

Physics-Based Models of Hysteresis in Multiphase Flow in Porous Media

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Conventional continuum simulation of immiscible two-phase flow in porous media is based on the following set of equations:

$$\phi \frac{\partial s_w}{\partial t} = -\nabla \cdot \mathbf{q}_w, \quad \phi \frac{\partial s_n}{\partial t} = -\nabla \cdot \mathbf{q}_n,$$
$$\mathbf{q}_w = -\frac{k_s k_{rw}(s_w)}{\mu_w} \left(\nabla p_w - \rho_w \mathbf{g} \right), \quad \mathbf{q}_n = -\frac{k_s k_{rn}(s_w)}{\mu_n} \left(\nabla p_n - \rho_n \mathbf{g} \right),$$
$$p_n - p_w = p_c(s_w),$$
$$s_w + s_n = 1,$$

where s_w and s_n denote the saturations of the wetting and nonwetting phases (each defined as the volume fraction of the pore space filled with the corresponding fluid), and p_w and p_w their respective pressures. In this formulation, the "capillary pressure", p_c, and the "relative permeabilities", k_{rw} and k_{rn}, are supplied as functions of s_w, typically via empirical constitutive relationships, but suffer from hysteresis – that is, p_c, k_{rw}, and k_{rn} may take on different values depending on the local history of s_w.

The morphology of the pore space – including the geometries of individual pore elements as well as their topological arrangement – is largely responsible for the observed hysteresis, but its effects may be challenging to describe at the continuum scale. Concepts like the pore-size distribution are useful for specifying the connection of effective pore radii to capillary equilibria and hydraulic conductances, but are ordinarily associated with the simplistic assumption that the pore space constitutes a bundle of straight, cylindrical capillary tubes, which neglects the network connectivity of different-radius pores.

In this thesis, we have established a new probabilistic framework for connecting the relevant microscopic properties and states of porous media to macroscopic quantities of interest in multiphase flow and other physical phenomena. The framework conceptualizes the pore space as an ensemble of pore-space instances, each formed by a number of independent homogeneous Poisson point processes corresponding to pore-scale events such as branching, pore-radius variation, and meniscus pinning.

We have then proposed the pore-space accessivity, $\alpha \in (0, 1)$, to characterize the arrangement of different-radius pore slices found in a porous sample. Its two limiting values, $\alpha \to 0$ and $\alpha \to 1$, correspond to different-radius pores being arranged overwhelmingly in series and overwhelmingly in parallel, the latter echoing the capillary bundle model. A lower accessivity corresponds to more prominent connectivity effects and leads to hysteresis in multiphase flow.

We have demonstrated that it is possible to measure the accessivity of a porous specimen by subjecting it to mercury cyclic porosimetry, a common characterization technique for porous materials. The formulae that we have proposed also yield a better estimate the pore-size distribution calculated from mercury porosimetry measurements compared to standard methods, which are known to overestimate the volume fraction of pores with smaller effective radii.

Having recognized that the conventional saturation variables, s_w and s_n, do not adequately describes the microscopic states of fluid phases in porous media, during multiphase flow, we have introduced the radius-resolved saturations, $\psi_w(F)$ and $\psi_n(F)$, to describe the pore-scale distribution of phases across different pore sizes, where F denotes the cumulative distribution function of the pore-size distribution. While $\psi_w(F)$ and $\psi_n(F)$ readily integrate to conventional saturations, they are distinguished in their inherent ability to predict hysteresis phenomena when used as continuum state variables. With the aid of our probabilistic framework, we have arrived at a series of relatively simple but physically meaningful formulae for hysteresis in capillary pressure and relative permeabilities. The formulae have been successfully implemented as hysteresis-enabling constitutive relationships in continuum simulations of multiphase flow.