Optimal Control of Dynamical Systems with Time-invariant Probabilistic Parameter Uncertainties

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The importance of taking model uncertainties into account during controller design is well established. Although this theory is well developed and quite mature, the worst-case uncertainty descriptions assumed in robust control formulations are incompatible with the uncertainty descriptions generated by commercial model identification software that produces time-invariant parameter uncertainties typically in the form of probability distribution functions. This doctoral thesis derives rigorous theory and algorithms for the optimal control of dynamical systems with time-invariant probabilistic uncertainties.

The main contribution of this thesis is new feedback control design algorithms for linear time-invariant systems with time-invariant probabilistic parametric uncertainties and stochastic noise. The originally stochastic system of equations is transformed into an equivalent deterministic system of equations using polynomial chaos (PC) theory. In addition, the $\mathcal{H}_2$- and $\mathcal{H}_\infty$-norms commonly used to describe the effect of stochastic noise on output are transformed such that the eventual closed-loop performance is insensitive to parametric uncertainties. A robustifying constant is used to enforce the closed-loop stability of the original system of equations. This approach results in the first PC-based feedback control algorithm with proven closed-loop stability, and the first PC-based feedback control formulation that is applicable to the design of fixed-order state and output feedback control designs. The numerical algorithm for the control design is formulated as optimization over bilinear matrix inequality (BMI) constraints, for which commercial software is available. The effectiveness of the approach is demonstrated in two case studies that include a continuous pharmaceutical manufacturing process.

In addition to model uncertainties, chemical processes must operate within constraints, such as upper and lower bounds on the magnitude and rate of change of manipulated and/or output variables. The thesis also demonstrates an optimal feedback control formulation that explicitly addresses both constraints and time-invariant probabilistic parameter uncertainties for linear time-invariant systems. The $\mathcal{H}_2$-optimal feedback controllers designed using the BMI formulations are incorporated into a fast PC-based model predictive control

(MPC) formulation. A numerical case study demonstrates an improvement in constraint satisfaction compared to past PC-based formulations for MPC.

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